Relationship of the spectrum of sea level muons to that of the primary cosmic rays

1974 J. Phys. A: Math. Nucl. Gen. 7 2074

(http://iopscience.iop.org/0301-0015/7/16/011)

J. Phys. A: Math., Nucl. Gen., Vol. 7, No. 16, 1974. Printed in Great Britain. © 1974

Relationship of the spectrum of sea level muons to that of the primary cosmic rays

A D Erlykin†, L K Ng‡ and A W Wolfendale

Physics Department, University of Durham, South Road, Durham DH1 3LE, UK

Received 24 April 1974

Abstract. An analysis is presented of the use of the spectrum of near vertical cosmic ray muons in deriving the energy spectrum of primary cosmic ray nucleons. The model used for high energy interactions in the atmosphere is that derived from ISR measurements, which cover the range $2{\cdot}5\times10^2$–$1{\cdot}5\times10^3$ GeV, and their extrapolation. The effect of variations from straightforward extrapolation is also examined.

The nucleon spectrum is used to derive alternative spectra of primary nuclei for different assumptions about the primary mass composition.

An analysis is also given of the distribution of primary nucleon energies and primary rigidities which contribute to muons of various threshold energies.

1. Introduction

In the previous paper (Erlykin *et al* 1974, to be referred to as I) an examination was made of the charge ratio of near vertical muons and its relevance to the form of the interaction of high energy nucleons with air nuclei. It was shown that for primary nucleon energies of several hundred GeV the results were not inconsistent with the hypothesis of limiting fragmentation. This hypothesis and the associated scaling hypothesis can be allowed to the very highest energies if the flux of heavy nuclei in the primary radiation increases somewhat with energy at energies above those at which direct measurements have been made.

In the present work we use the experimental data on the muon spectrum, as distinct from the charge ratio, to derive information about the primary spectrum. Such a procedure is not new; for example Brooke *et al* (1964) used the data then available on both the muon and proton spectra, together with a model for high energy interactions which was valid at some tens of GeV, to estimate the primary nucleon spectrum. The result had value in that it showed that the spectrum was somewhat steeper than had previously been assumed and lent support to the contention that there was a rather rapid change in the exponent at an energy somewhat above 10^5 GeV. The present work essentially represents an updating of this work. A similar treatment has recently been made by Ramana Murthy and Subramanian (1972).

The question of the primary nucleon energies responsible for muons of particular energies is of importance for a consideration of the nuclear physics of the subject and this will be discussed. Of greater interest is the distribution of primary rigidities

---

† On leave from Lebedev Physical Institute, Moscow.

‡ On leave from University of Hong Kong, Hong Kong.

responsible for muons above specific threshold energies in view of the interpretation of measurements of the variation with sidereal time of the muon rate. The point is that 'low' rigidity primaries are subject to deflexions in the magnetic field associated with the solar wind which nullify the anisotropies (if any) of arrival directions of the galactic cosmic rays. The search for such anisotropies is of great importance in attempting to answer the basic question of whether or not the cosmic rays of the appropriate energies are derived from galactic sources or whether they arise from outside the galaxy.

2. The muon spectrum

The energy spectrum of near vertical muons is a 'constant' of the cosmic radiation, at least for energies above 10 GeV or so, and it has been the subject of considerable experimental study. Various methods have been employed, primarily the use of magnetic spectrographs at energies below some hundreds of GeV (with which muon momenta are determined directly), the analysis of 'bursts' in local absorbers in the range 10^3–10^4 GeV and derivation from the measured variation of muon intensity with depth underground from several GeV to somewhat above 10^4 GeV.

A survey has been made recently by Ng and Wolfendale (1974) and this will be adopted here. Although one of the latest summaries it is still not to be regarded as of high accuracy throughout—better measurements are still needed and some are in fact currently being made.

The integral spectrum from this work is summarized in figure 1; to these measurements have been added, at the lower energy end, recent results from the Durham spectrograph MARS (Ayre *et al* 1973). The final adopted differential muon spectrum has the intensities given in table 1.

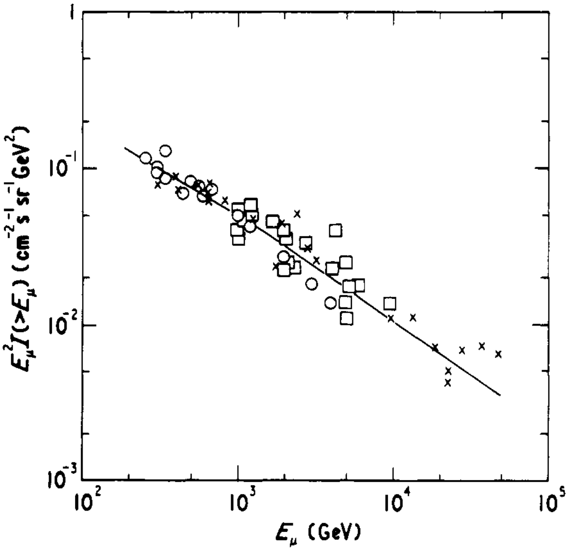

Figure 1. Adopted integral spectrum of near vertical muons from the summary of Ng and Wolfendale (1974). The symbols denote the types of data used in the summary: ○ magnetic spectrographs and atmospheric γ-cascade analyses; □ measurements on bursts in local absorbers; × depth–intensity measurements. Indications of the accuracy of the various intensities are not given; they vary considerably but typical errors are ±30%.

The line is a best fit to the data assuming a constant exponent of the pion production spectrum, integral value $\gamma_\pi = 1{\cdot}70 \pm 0{\cdot}03$.

Table 1. Adopted muon intensities.

Muon energy (GeV)	20	50	100	200	500	1000
Intensity ($cm^{-2}\,s^{-1}\,sr^{-1}\,GeV^{-1}$)	$2{\cdot}5\times10^{-5}$	$2{\cdot}1\times10^{-6}$	$3{\cdot}0\times10^{-7}$	$3{\cdot}4\times10^{-8}$	$1{\cdot}7\times10^{-9}$	$1{\cdot}35\times10^{-10}$

Muon energy (TeV)	2	5	10	20	50	100
Intensity ($cm^{-2}\,s^{-1}\,sr^{-1}\,GeV^{-1}$)	$1{\cdot}04\times10^{-11}$	$3{\cdot}5\times10^{-13}$	$2{\cdot}7\times10^{-14}$	$2{\cdot}06\times10^{-15}$	$6{\cdot}96\times10^{-17}$	$5{\cdot}4\times10^{-18}$

3. Derivation of the primary nucleon spectrum

In I a summary of the relevant ISR data was given and the relationship between the primary nucleon spectrum and the muon spectrum was given in some detail. The analysis used there has been used to give the respective contributions to the muon flux from the various interactions; these are shown in figure 2. The ordinate represents the ratio of the intensity of muons to nucleons of the same energy. Insofar as the secondary particle spectra are weighted due to the form of the primary spectrum the results are

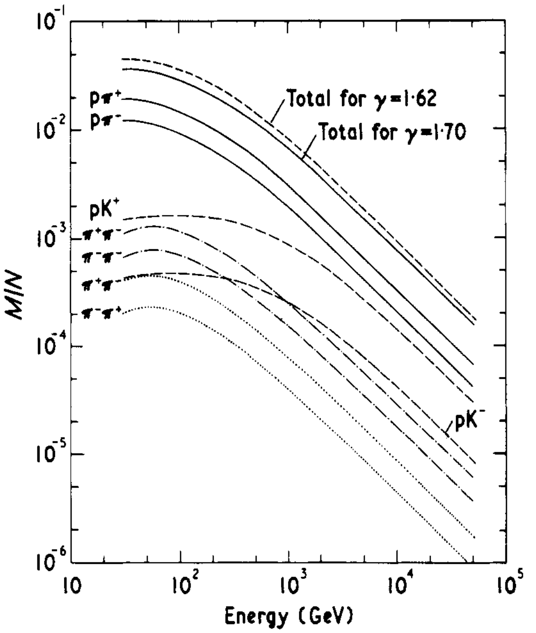

Figure 2. Ratio of muon to primary proton intensities derived from data given in the previous paper (Erlykin *et al* 1974) for $\gamma = 1{\cdot}70$. The total, alone, is given for $\gamma = 1{\cdot}62$; the constituents are scaled down by the same factor. The symbols represent the interaction processes responsible (eg pK^- indicates muons from K^- mesons generated in proton–air nucleus interactions). The curves for primary neutrons are identical in magnitude but reversed in sign for secondary pions (ie $p\pi^+ \equiv n\pi^-$). The curves for secondary kaons from neutron primaries are taken to be the averages for pK^+ and pK^- (see I). The contributions from secondary pion interactions (eg $\pi^+\pi^-$) are also shown.

dependent on the exponent of the primary spectrum but this dependence is not very great (see table 2 in I). For example, for the important process of positive pion production by nucleons the values of $\langle nx^{\gamma} \rangle$ (the quantity to which the muon intensity is proportional) is 0·041 for $\gamma = 1{\cdot}6$ and 0·035 for $\gamma = 1{\cdot}7$; here γ is the exponent of the integral primary spectrum. In figure 2 the constituent components, and the total, for $\gamma = 1{\cdot}70$ are shown and the total alone is indicated for $\gamma = 1{\cdot}62$. As was indicated in I, the spectral exponent appears to be near 1·62 for primary energies below about 10^3 GeV/nucleon and about 1·70 for energies above this value, although as was indicated these values are not firm.

Application of the M/N ratio in figure 2 to the muon spectrum of figure 1 gives the primary nucleon spectrum shown in figure 3.

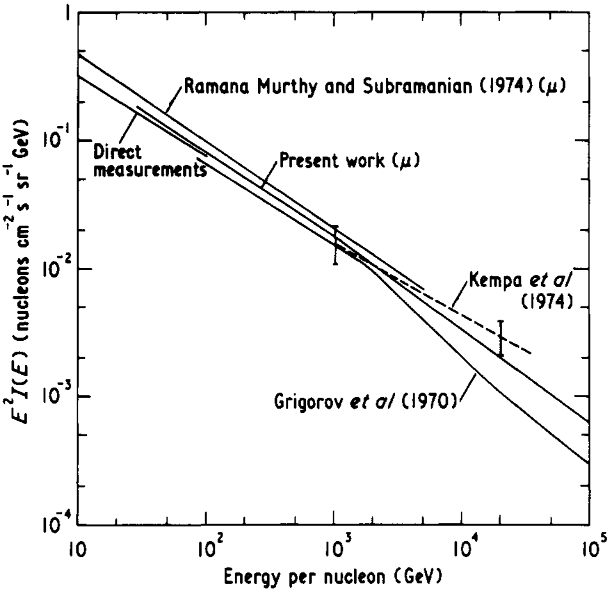

Figure 3. Differential energy spectrum of primary nucleons. μ denotes primary spectra derived from sea level muon spectra. The other spectra are described in the text.

As remarked in I, the near agreement of the measured and predicted muon charge ratios indicates that the effect of intranuclear cascading on the spectrum of energetic secondaries from nucleon–air nucleus interactions is small, that is, the spectrum is well represented by the ISR data for p–p collisions. This means that the calculated primary nucleon spectrum (figure 3) is probably rather accurate below about 10^3 GeV. At higher energies the calculation corresponds to the assumption that the interaction characteristics remain unchanged, ie that intranuclear cascading continues to be negligible and that the scaling hypothesis continues to be valid for two more decades. In fact, Wdowczyk and Wolfendale (1973) contend that there may be an increasing breakdown of scaling above 10^3 GeV, a suggestion which arises from the fact that at much higher energies the effective multiplicity of secondaries in particle–air nucleus collisions is much higher than expected from the scaling idea (which predicts a logarithmic dependence on energy of the total multiplicity and a near constant multiplicity of those secondaries which carry the bulk of the energy). However, the operative word in the foregoing is ‘particle’. Only if the primary particles are protons is a breakdown of scaling demanded; as the authors point out, if the primaries are heavy nuclei, the validity of scaling can be restored. The derived nucleon spectrum above 10^3 GeV is thus to be regarded as a lower limit since, if the particles are mainly protons and the secondaries are more numerous than scaling indicates, the average energy per secondary will fall and with it the ratio M/N.

Also shown in figure 3 is the prediction from a similar analysis by Ramana Murthy and Subramanian (1972). The difference from the present work (20–30%) is easily understood in terms of the adoption of a slightly different muon spectrum and changes in the ISR data from the data of the earlier work to the present time.

The results of direct measurements, summarized in I, are shown in figure 3 for the energy range where such measurements are reasonably precise. The closeness to the spectrum from the muon analysis (within about 10%) is very satisfactory and adds confidence to the analysis.

The direct measurements of the primary spectrum which extend to the highest energies are those of the PROTON series of satellites (Grigorov *et al* 1970). These measurements have aroused controversy in that they appear to indicate that the proton spectrum falls off rather rapidly above about 10^3 GeV, whereas the spectrum of heavier nuclei continues with about the same exponent. The argument against these results has been that EAS measurements at higher energies ($\gtrsim 10^6$ GeV) appear to show fluctuations in various parameters indicative of the presence of a significant fraction of protons (see, for example, the reviews by Trümper (1970) and Thompson *et al* (1970)) and they are not inconsistent with the continuation of a 'normal' composition, ie the composition pertaining to approximately 10 GeV/nucleon. Such arguments are attractive but not compelling in view of the fact that if cosmic rays of different energies come from sources of different types then the mass composition may in fact be a function of energy. Indeed the recent measurements of Ormes *et al* (1973) and others show a change in composition over quite a small range of energy (10–100 GeV/nucleon) and, although this may be due merely to a change of galactic propagation characteristics and be of transitory existence, it could conceivably indicate a continuing trend of primary mass. The data of Grigorov *et al* (1970) on the spectrum of all nuclei and that of protons alone have been used to give the spectrum of nucleons and this is shown in figure 3. It is seen that the intensities become increasingly lower than our prediction as energy increases and insofar as our intensities above 10^3 GeV/nucleon are lower limits there appears to be an inconsistency. However, it must be remarked that in our calculations we have assumed that a heavy nucleus of mass A behaves as A independent nucleons in its interactions in the atmosphere. This is clearly an approximation and it is possible that the muon yield is somewhat in excess of our predictions; this would, in turn, lower the primary spectrum derived from the measured muon spectrum.

It is possible that the mass composition indicated by the PROTON measurements is correct, whether the spectral shape is correct or not, and the consequences of taking the composition from these data will be examined later.

A recent survey of data on the primary spectrum has been made by Kempa *et al* (1974) and the results for primary nucleons are indicated in figure 3. The spectrum in this region is derived from the analysis by Kempa (1973) of the propagation of the nucleonic component in the atmosphere. There is seen to be reasonable agreement with our prediction within the rather large errors.

4. The spectrum of primary nuclei

Conversion from the spectrum of nucleons to nuclei is straightforward if the composition and the exponents of the constituent spectra are known. Table 2 shows the conversion

Table 2. Conversion factors for primary intensities (using the data from I, which relate to $E < 100$ GeV/nucleon).

$\frac{\text{Nucleons}}{\text{Protons}}$	$\frac{\text{Nuclei}}{\text{Protons}}$	$\frac{\text{Nuclei}}{\text{Nucleons}}$
1·37	2·40	1·75

factors for the simple case of constant composition (with the composition indicated in I for $E < 100$ GeV/nucleon) and exponent $\gamma = 1{\cdot}62$.

Figure 4 shows the primary energy spectra calculated from the spectrum of nucleons (derived from the muon data) under a variety of assumptions about the primary composition. Also indicated is the lower limit to the intensity derived by Wdowczyk and Wolfendale (1973) from EAS data (calculated assuming primary protons). The more recent analysis by Kempa *et al* (1974) gives the curve indicated; this is a composite curve through the measurements of Kempa (1973) (converted from nucleons to nuclei) which relate to $E_p < 5 \times 10^4$ GeV and the summarized EAS data. The intensities are seen to be higher than given by the derivation of Wdowczyk and Wolfendale above 4×10^5 GeV; this arises because the analysis of Kempa *et al* uses data from a variety of experiments whereas the earlier analysis used only the measurements of Bradt *et al* (1966). In fact, all the EAS measurements may be in error and both intensities may be overestimates although this seems rather unlikely. We regard the most likely spectrum (assuming constant composition) and the conventional model for the interaction of nuclei (see § 3) to be between the two, but closer to the summary of Kempa *et al.*

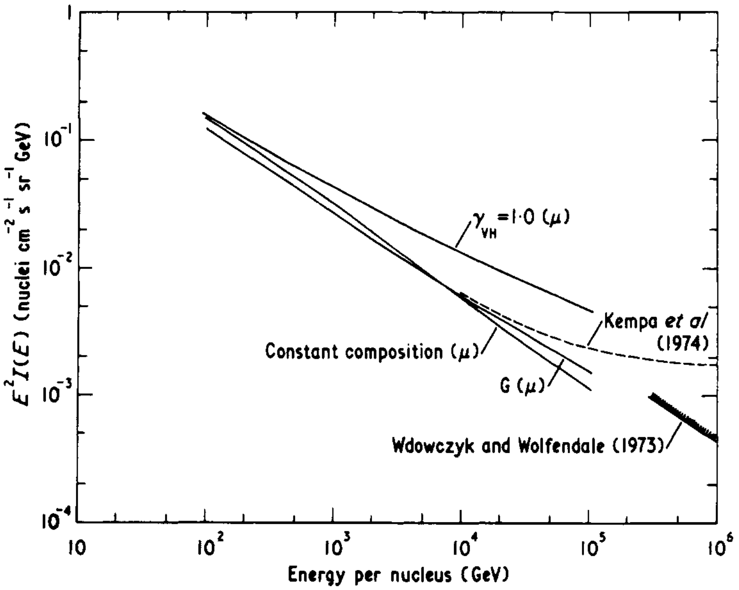

Figure 4. Differential energy spectrum of primary nuclei. μ denotes spectra derived from the sea level muon spectrum adopted in the present work under differing assumptions about the primary mass composition. $\gamma_{VH} = 1{\cdot}0$ corresponds to a primary iron spectrum with this exponent, constant composition indicates the same composition as measured at 10 GeV/nucleon and G denotes the mass composition given by Grigorov *et al* (1970).

The spectra derived from muons correspond to the case of constant mass composition, the Grigorov-type composition, in which the proton intensity falls very rapidly above 10^3 GeV and the limiting composition of I, in which the iron spectrum has a very flat spectrum (integral exponent $\gamma_{\mathrm{VH}} = 1{\cdot}0$). If the primaries are indeed largely heavy nuclei at about 10^6 GeV/nucleus then the spectral intensities related to EAS data will be a little higher than indicated in figure 4 because the EAS measurements contributing to the intensities in this region come from electron shower sizes and as the mass increases the size falls for the same primary energy per nucleus.

The conclusion to be drawn from figure 4 is that an extrapolation of the spectrum derived for constant composition falls by a factor of rather more than 2 below the intensities from the EAS data. If there is really an inconsistency then two possibilities arise: (i) a breakdown in scaling above 10^3 GeV/nucleon with no change in composition, or (ii) an increase in the fraction of heavy nuclei in the primary beam. An extrapolation of the curve $\gamma_{\mathrm{VH}} = 1{\cdot}0$ would appear to give intensities a little too high compared with the EAS intensities and, as was noted in I, there is some information there too that this value of γ is a little too low. All the data would probably be better fitted with an integral spectral index for the iron nuclei a little greater than 1·0 beyond approximately 10^4 GeV/nucleon.

5. Response function for various muon threshold energies

5.1. *Response function in terms of primary energy per nucleon*

5.1.1. Calculations from the present work. It was pointed out in § 1 that a knowledge of the distribution of energy of primary particles giving rise to muons of energy above some limiting value is of nuclear physical interest and the ensuing distribution of rigidities is a prerequisite for the interpretation of the siderial variation of the muon flux. The basic ideas for the derivation of these quantities are given in I, ie the energy spectra of secondary particles from the various interactions ($\mathrm{d}\sigma/\mathrm{d}x$ for $\mathrm{p}\pi^+$, $\mathrm{p}\pi^-$ etc) and the expressions which give the relative contributions from the successive interactions of primary nucleons and secondary pions. For a particular channel, eg $\mathrm{p}\pi^+$, the fraction of interacting nucleons having energy less than E_{p} giving rise to parent pions having energy E_π (the 'response function' for E_π) is given by

$$P[E_{\mathrm{p}}, E_\pi]_{\mathrm{p}\pi^+} = \int_x^1 x^\gamma \left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_{\mathrm{p}\pi^+} \mathrm{d}x,$$

where $x = E_\pi/E_{\mathrm{p}}$.

In making an accurate derivation of the response function for muons of a fixed energy at ground level $P(E_{\mathrm{p}}, E_\mu)$ it is necessary to allow for all the various parents of the muons and, more particularly, the contributions to the muon flux of the various generations of parents produced in the atmosphere. For example, restricting attention to muons from parent pions, if the mean primary nucleon energy giving rise to first generation muons of energy E_μ is E_{p} then the primaries responsible for second generation muons have, approximately, mean energy E_{p}/α (where α is the elasticity), those of third generation have energy E_{p}/α^2 etc. The contributions from these energies to the muon flux are, respectively

$$D_1, \alpha^\gamma D_2, \alpha^{2\gamma} D_2, \ldots$$

where $D_1, D_2, \ldots$ are the mean probabilities of pions produced in the various generations giving rise to muons of the detected energies. (Typically, for 14 GeV muons, $D_2/D_1 = 0{\cdot}94, D_3/D_1 = 0{\cdot}89$.)

Calculations have been carried out using the appropriate propagation expressions indicated in I, and with the constants given there, to give the response functions for the various generations and for parent pions and kaons, with the result shown in figure 5. Also shown is the aggregate response function.

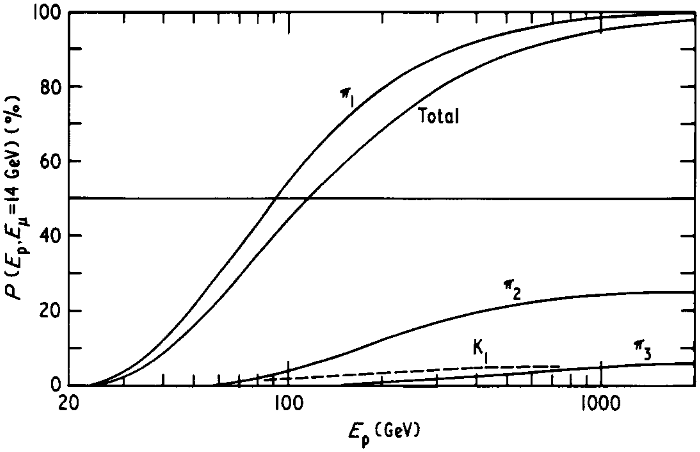

Figure 5. Contributions to the response function (in terms of primary energy per nucleon) from the various generations for near vertical muons of energy 14 GeV.

It will be appreciated that the functions are sensitive to the values of α and γ. Changing α from 0·45 (the present value) to 0·55 increases the value of E_p at which $D(E_p, E_\mu = 14\,\mathrm{GeV}) = 50\%$ by about 11 %, a change that arises because of the smaller effect of later generations. Reducing γ from 1·7 to 1·6 increases the same quantity by 20 %, a value that agrees with the calculations of Gaisser (1973, 1974).

Response functions for muons of energy above various muon threshold energies $P(E_p, > E_\mu)$ have been determined by deriving $P(E_p, E_\mu)$ for various E_μ and integrating over the differential muons energy spectrum. For the energies in question,

$$14 \lesssim E_\mu \lesssim 100\,\mathrm{GeV},$$

the differential muon spectrum is given to sufficient accuracy by the expression $N(E_\mu) \propto (E_\mu + \Delta E_\mu)^{-2{\cdot}97}$ with E_μ in GeV (the quantity ΔE_μ is the mean energy loss of muons, to the main production level, and is approximately 2·5 GeV).

The corresponding response functions are given in figure 6 for three muon threshold energies and figure 7 gives a universal function from which $P(E_p, > E_\mu)$ may be derived for intermediate energies.

Before comparing the results with those of other workers it is necessary to recapitulate some of the basic assumptions. It has been assumed that scaling exists over the whole energy range; this is reasonable in view of the ISR data covering most of the range and, as has been mentioned earlier there is evidence from the ISR experiments that scaling is valid for the important pion secondaries. It has been assumed that there is negligible intranuclear cascading; this is considered to be a tenable assumption because of the demonstration in I that such cascading is negligible at the relevant x values.

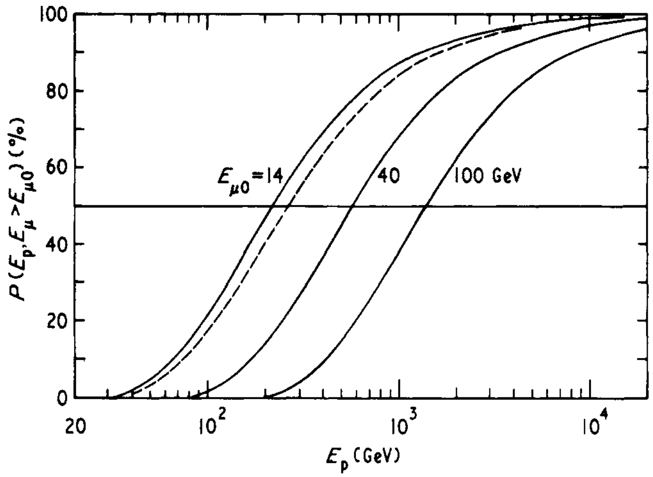

Figure 6. Response functions (in terms of primary energy per nucleon) for near vertical muons of energy above 14, 40 and 100 GeV. The broken line represents the response function in terms of rigidity for $E_\mu > 14$ GeV; for this function the horizontal scale is rigidity in GV.

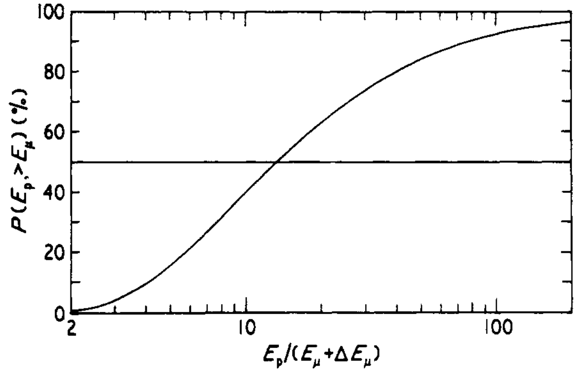

Figure 7. The response function (in terms of primary energy per nucleon) expressed in a general form. The curve is reasonably accurate for $14 \lesssim E_\mu \lesssim 100$ GeV.

5.1.2. Comparison with previous calculations. The most recent calculations are those of Gaisser (1973, 1974) and the response function derived by this author is shown in figure 8. Also shown there are functions from the survey of previous calculations given by Speller *et al* (1972) (see figure caption for key) and converted by Gaisser (1974) to standard conditions.

Some, at least, of the discrepancies can be understood. Thus, the function attributed to one of the present authors (Wolfendale 1970, quoted by Speller *et al* 1972) was derived under the assumption that the mean multiplicity of secondary pions varied with primary energy as $E_p^{1/4}$ whereas in the present work the scaling hypothesis with its near constant multiplicity is adopted. The difference is in the correct direction. With the unlikeliness of any appreciable intranuclear cascading at several hundred GeV/nucleon, and the information from ISR, the characteristics of interactions in this energy region are now reasonably well known and we must regard the present response function as more accurate than that calculated hitherto.

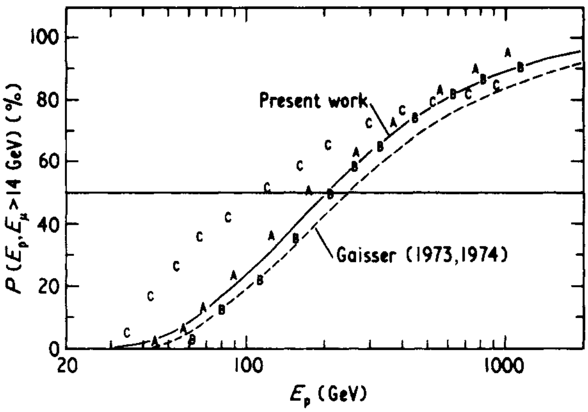

Figure 8. Response functions from various authors for $E_\mu > 14$ GeV: A, Peacock (1971); B, Wolfendale (1970); C, Ahluwalia and Ericksen (1971). The points denoted by A, B and C have been converted by Gaisser (1974) to correspond to the standard conditions of $E_\mu > 14$ GeV (or to be precise 13·9 GeV) and to relate to primary energy per nucleon. It should be noted that there are significant differences from the comparisons given in the earlier work of Gaisser (1973) (the authors are grateful to Dr Gaisser for information on this point).

The treatment of Gaisser is very similar to that in the present work; the response function indicated relates to a primary spectrum with integral exponent $\gamma = 1{\cdot}62$ and is therefore comparable; however, Gaisser takes a mean elasticity α of 0·5 compared with our value of 0·45 and this is responsible for much of the discrepancy. The sensitivity of $P(E_p, E_\mu)$ to α was indicated in § 5.1.1; increasing α from 0·45 to 0·5 in our own work increases the 50 % energy of figure 8 (ie the median energy) from 216 GeV to 230 GeV a value that is close to the 240 GeV of Gaisser.

We conclude that, taking into account all the uncertainties of which we are aware, the median primary nucleon energy for $E_\mu > 14$ GeV is probably 230 ± 30 GeV.

5.2. Response function in terms of primary rigidity

As was remarked in § 1, for astrophysical studies it is the primary rigidity, $pc/Ze(\simeq E/Z)$, rather than energy per nucleon that is of importance and the response function in terms of rigidity can be derived directly from what has been given in § 5.1. In I it was pointed out that for energies up to 100 GeV/nucleon, at least, the neutron to nucleon ratio is $\eta = 0{\cdot}136$; the corresponding ratio of nucleons in nuclei to protons alone is about $2\eta/(1-2\eta) = 0{\cdot}375$, assuming that $A = 2Z$ for the bulk of the nuclei with $A > 1$. The response function for rigidity follows directly by taking data of the type given in figure 6 weighted in asymptotic probability to correspond to the fractional proton flux and adding to it a response function displaced to twice the energy and weighted to correspond to the number of nucleons in nuclei. The function found in this way for $E_\mu > 14$ GeV is also shown in figure 6.

It has been assumed throughout that the composition is unchanged. If there is an increase in the relative flux of heavy nuclei above 100 GeV/nucleon then the approach to the asymptotic probability in figure 6 will be delayed somewhat and the median rigidity for $E_\mu > 14$ GeV will be increased.

The median rigidity for $E_\mu > 14$ GeV from figure 6 is 270 GV and, again, we estimate an uncertainty of about $\pm 15\%$.

Acknowledgments

The authors are grateful to the Science Research Council for continued support. Dr T K Gaisser and Mr C J Hume are thanked for useful discussions.

References

Ahluwalia H S and Erickson J H 1971 *J. Geophys. Res.* **76** 6613–27

Ayre C A *et al* 1973 *Proc. 13th Int. Conf. on Cosmic Rays, Denver* vol 3 (Denver: University of Denver) pp 1822–7

Bradt H *et al* 1966 *Proc. 9th Int. Conf. on Cosmic Rays, London* vol 2 (London: The Institute of Physics and the Physical Society) pp 715–7

Brooke G, Hayman P J, Kamiya Y and Wolfendale A W 1964 *Proc. Phys. Soc.* **83** 853–69

Erlykin A D, Ng L K and Wolfendale A W 1974 *J. Phys.* A: *Math., Nucl. Gen.* **7** 2059–73

Gaisser T K 1974 *J. Geophys. Res.* in the press

—— 1973 *Proc. 13th Int. Conf. on Cosmic Rays, Denver* vol 3 (Denver: University of Denver) pp 1860–5

Grigorov N L *et al* 1970 *Sov. J. Nucl. Phys.* **11** 588–94

Kempa J 1973 *PhD Thesis* University of Lodz, Lodz, Poland

Kempa J, Wdowczyk J and Wolfendale A W 1974 *J. Phys.* A: *Math., Nucl. Gen.* **7** 1213–21

Ng L K and Wolfendale A W 1974 *Nuovo Cim.* B **20** 161–72

Ormes J F, Balasubrahmanyan V K and Arens J F 1973 *Proc. 13th Int. Conf. on Cosmic Rays, Denver* vol 4 (Denver: University of Denver) pp 157–62

Peacock D S 1969 *Proc. 11th Int. Conf. on Cosmic Rays, Budapest* vol 2 (Budapest: Akademii Kiado) pp 189–94

Ramana Murthy P V and Subramanian A 1972 *Phys. Lett.* **39B** 646–8

Speller R, Thambyahpillai T and Elliot H 1972 *Nature, Lond.* **235** 25–9

Trümper J 1970 *Proc. 6th Interam. Sem. on Cosmic Rays, La Paz* vol 2 (San Andreas: University of San Andreas) pp 372–92

Thompson M G, Turner M J L, Wdowczyk J and Wolfendale A W 1970 *Acta Phys. Acad. Sci. Hung.* **29** *Suppl.* 3, 615–20

Wdowczyk J and Wolfendale A W 1973 *J. Phys.* A: *Math., Nucl. Gen.* **6** 1594–611